

EXHIBIT V

1.4.1 Median Willingness to Pay
 The Turnbull provides an estimate of the range in which median willingness to pay falls. Since the no-response proportions are consistent estimates of the distribution point masses at each price, the price for which half of the distribution function passes 0.5 is the lower bound on the range of the distribution function. The next highest price represents the upper bound on median WTP. For example, if 30% of respondents say no to the range of median WTP \$15-\$20. The median represents the price for which the probability of a no response equals 0.5. Since the Turnbull only gives point mass estimates at a discrete number of points, the median can only be defined within a range.

3.4.2 A Lower Bound Estimate for Willingness to Pay

A Single Price Case

For simplicity, consider a case in which all individuals are offered the same price t . In this case, a conservative estimate of expected willingness to pay will be the product of the offered price and the probability of willingness to pay being above the price: $t \cdot (1 - F(t))$. To see why this is a conservative estimate consider the general expression for the expected value of the random variable WTP, assumed to be distributed between 0 and U :

$$E(WTP) = \int_0^U W dF_W(W), \quad (3.21)$$

where U is the upper bound on the range of WTP. By partitioning the range of willingness to pay into two sub-ranges according to the offered price $[0, t]$, and $[t, U]$, the expected value of willingness to pay can be written

$$E(WTP) = \int_0^t W dF_W(W) + \int_t^U W dF_W(W). \quad (3.22)$$

Because $F_W(W)$ is a cumulative distribution function it is increasing. Hence replacing the variable of integration by the lower limit will result in an expression less than or equal to $E(WTP)$, that is, a lower bound on willingness to pay

$$E(WTP) \geq \int_0^t W dF_W(W) + \int_t^U t dF_W(W) = t \cdot (1 - F_W(t)). \quad (3.23)$$

The equality holds by assuming that $F_W(U) = 1$. The expression states that expected willingness to pay is at least as great as the offered price

multipled by the probability of a yes response to the offered price. For example, if the offered price is \$10 and the sample probability of a yes response is 0.25, then expected willingness to pay must be at least \$2.50.

We can identify this measure of willingness to pay as $E_{LB}(WTP) = t \cdot (1 - F_W(t))$, the lower bound on expected willingness to pay. Substituting in the Turnbull estimate for $F_W(t)$, we obtain a consistent estimate of the lower bound on expected willingness to pay

$$E_{LB}(WTP) = t \cdot \frac{Y}{T}. \quad (3.24)$$

The Multiple Price Case

A similar procedure first employed by Carson, Hanemann et al. (1994) can be used to define a lower bound on willingness to pay when M^* distinct prices are randomly assigned to respondents. (Throughout this section we use the notation f_j^* and F_j^* . These refer to the pooled data if the original data are not monotonic. Otherwise we refer to the original data.) Recall the definition of willingness to pay from equation (3.21)

$$E(WTP) = \int_0^U W dF_W(W) \quad (3.25)$$

where U is the upper bound on the range of WTP. The range of willingness to pay can now be divided into $M^* + 1$ subranges: $\{0 = t_1, t_1 - t_2, \dots, t_{M^*} - U\}$. Using these ranges, expected willingness to pay can be written as

$$E(WTP) = \sum_{j=0}^{M^*} \left[\int_{t_j}^{t_{j+1}} W dF_W(W) \right] \quad (3.26)$$

where $t_0 = 0$, and $t_{M^*+1} = U$. Because $F_W(W)$ is an increasing function, we know that $\int_{t_{j+1}}^{t_{j+2}} W dF_W(W) \geq \int_{t_j}^{t_{j+1}} t_j dF_W(W)$. Hence we can write

$$E(WTP) \geq \sum_{j=0}^{M^*} t_j [F_W(t_{j+1}) - F_W(t_j)] \quad (3.27)$$

where we use $\int_{t_j}^{t_{j+1}} t_j dF_W(W) = t_j \{F_W(t_{j+1}) - F_W(t_j)\}$. For calculating this sum, one needs the results $F_W(0) = 0$ and $F_W(U) = 1$. Substituting in the consistent estimator for $F_W(t_j)$, and simplifying notation so $F_W(t_j) = F_j^*$, a consistent estimate of the lower bound on willingness to pay is

$$E_{LB}(WTP) = \sum_{j=0}^{M^*} t_j (F_{j+1}^* - F_j^*) \quad (3.28)$$

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where $F_j^* = \frac{N^*}{T_j}$, $F_0^* = 0$, and $F_{M^*+1}^* = 1$. This lower bound estimate of willingness to pay has an intuitive interpretation. By multiplying each offered price by the probability that willingness to pay falls between that price and the next highest price, we get a minimum estimate of willingness to pay. The estimated proportion of the sample that has willingness to pay falling between any two prices is assumed to have willingness to pay equal to the lower of those two prices. This estimate is appealing because it offers a conservative lower bound on willingness to pay for all non-negative distributions of WTP, independent of the true underlying distribution. Even though the true distribution of willingness to pay is unknown, $E_{LB}(WTP)$ will always bound expected willingness to pay from below as long as the true distribution of willingness to pay is defined only over the non-negative range. In practice, $E_{LB}(WTP)$ represents the minimum expected willingness to pay for all distributions of WTP defined from zero to infinity.

Using a similar procedure, an upper bound on willingness to pay can be defined as: $E_{UB}(WTP) = \sum_{j=0}^{M^*} t_{j+1}^* (F_{j+1}^* - F_j^*)$. The problem here, however, lies in the definition of t_{M^*+1} . Since p_{M^*} is the highest offered bid it is necessary to define the upper bound on the range on willingness to pay using an ad hoc method. Income is a possibility, but income can lead to large estimates of the upper bound. Other measures are possible but will be difficult to defend against a charge of being arbitrary.

One advantage of the lower bound estimate of WTP is the distribution of the estimator. Since the f_j^* 's are normal and the t_j 's are fixed, the $E_{LB}(WTP)$ is also normal. Normality makes its variance worth computing. Rewriting the expected lower bound in terms of the probability mass estimates gives

$$E_{LB}(WTP) = \sum_{j=0}^{M^*} t_j \cdot f_{j+1}^*. \quad (3.29)$$

The variance of the lower bound estimate is

$$V(E_{LB}(WTP)) = \sum_{j=0}^{M^*} t_j^2 V(f_{j+1}^*) + \sum_{i=1}^{M^*} \sum_{j=i}^{M^*} t_j t_{j+1}^* cov(f_j^*, f_{j+1}^*). \quad (3.30)$$

Recalling that $V(f_j^*) = V(F_j^*) + V(F_{j-1}^*)$, and

$$cov(f_i^*, f_j^*) = \begin{cases} -V(F_i^*) & j-1 = i \\ -V(F_j^*) & i-1 = j \\ 0 & \text{otherwise.} \end{cases}$$

The variance of the expected lower bound simplifies to

$$V(E_{LB}(WTP)) = \sum_{j=1}^{M^*} \frac{F_j^*(1-F_j^*)}{T_j} (t_j - t_{j-1})^2 \quad (3.31)$$

$$= \sum_{j=1}^{M^*} V(F_j^*) (t_j - t_{j-1})^2. \quad (3.32)$$

The variance can be used for constructing hypothesis tests and confidence intervals about $E_{LB}(WTP)$. Because $E_{LB}(WTP)$ is a linear function of the asymptotically normal maximum likelihood distribution function estimates f_j^* , $E_{LB}(WTP)$ will be normally distributed with mean defined in equation (3.28) and variance defined in equation (3.31):

$$E_{LB}(WTP) \sim N \left(\sum_{j=0}^{M^*} t_j (F_{j+1}^* - F_j^*), \sum_{j=1}^{M^*} \frac{F_j^*(1-F_j^*)}{T_j} (t_j - t_{j-1})^2 \right). \quad (3.33)$$

Procedure for Computing Lower Bound Willingness to Pay From a CV Survey with Multiple Prices

1. Calculate the proportion of no responses to each offered price by dividing the number of no responses by the total number of respondents offered each price. Denote this F_j^* . These are derived from pooling if necessary. Recall that $F_0^* = 0$ and $F_{M^*+1}^* = 1$. These represent consistent estimates of the probability of a no response to each offered price.

2. Calculate $f_{j+1}^* = F_{j+1}^* - F_j^*$ for each price offered. These represent consistent estimates of the probability that willingness to pay falls between price j and price $j+1$. To calculate the probability that willingness to pay is between the highest bid (t_M) and the upper bound (t_{M+1}), we define $F_{M^*+1} = 1$. This means that no respondents have willingness to pay greater than the upper bound.

3. Multiply each offered price (t_j) by the probability that willingness to pay falls between it and the next highest price (t_{j+1}) from step 2. We do not need to perform this calculation for the interval $0 - t_1$ since it entails multiplying the probability by zero.

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cent think of this estimate as the sum of marginal values times quantity adjustments, or the integral over quantity in a demand curve.

5. Calculate the variance of the lower bound as

$$V(E_{LB}(WTP)) = \sum_{j=1}^{M^*} \frac{F_j^*(1 - F_j^*)}{T_j^*} (t_j - t_{j-1})^2$$

Example 8 Willingness to Pay Estimates for the Turnbull

In a test of real versus hypothetical responses to hypothetical referenda, Cummings et al. (1997) perform an experiment in which one random sample is offered a single hypothetical payment (t) of \$10.00 to provide a good, and a second sample is made the same offer except told the payment of \$10.00 is real. Their results are summarized in Table 3.4. Because only a single bid was offered, the Turnbull estimate of the payment for a single price

	Hypothetical	Real	Total
No	102	73	175
Yes	84	27	111
Total	186	100	286

TABLE 3.4. Hypothetical and Real Responses for a Single Price

the probability of a no response is simply the proportion of respondents responding no to that bid. The probability of a no response can be estimated as: $F(\$10) = 102/186 = 0.548$ for the hypothetical experiment, and $F(\$10) = 73/100 = 0.730$ for the real experiment. The respective variances are: $V(F(\$10)) = 0.01133$ for the hypothetical responses and $V(F(\$10)) = 0.00197$ for the real responses. The t-statistics for significant difference from zero are 16.03, and 16.45 respectively. The t-statistic for difference in means is 3.17.³

A lower bound estimate of willingness to pay is found by multiplying the offered price by the estimate of the probability of a yes ($1 - F(\$10)$). For the hypothetical data, expected willingness to pay is $\$10 \cdot (1 - 0.548) = \4.52 with an estimated variance of 0.133. For the real data, the lower bound estimate of expected willingness to pay is $\$10 \cdot (1 - 0.73) =$

³The test statistic for difference in means is: $t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\sigma_1^2 + \sigma_2^2}}$, where μ is the relevant mean, σ^2 is the relevant variance.

$\$2.70$ with an estimated variance of 0.197 . To test the significance of the difference between the two means the standard test for the significance of the difference between two normally distributed variables can be used. The t-statistic is $(\$4.52 - \$2.70)/\sqrt{0.133 + 0.197} = 3.17$. This simple test rejects the hypothesis that willingness to pay in the real and hypothetical experiments is equal.

In a study with several bid prices, Duffield reports the results of a contingent valuation survey to value wolf recovery in Yellowstone National Park. The recorded responses are summarized in the first three columns of Table 3.5.

TABLE 3.5. Turnbull Estimates with Pooling

t_j	N_j	T_j	Unrestricted		Turnbull	
			P_j	F_j^*	P_j	f_j^*
5	20	54	0.370	0.343	0.343	0.343
10	15	48	0.313	Pooled back	Pooled back	Pooled back
25	46	81	0.568	0.568	0.568	0.568
50	65	96	0.579	0.579	0.579	0.579
100	106	133	0.797	0.797	0.797	0.797
200	82	94	0.872	0.872	0.872	0.872
300+	72	81	0.889	0.889	0.889	0.889
	-	-	1	1	1	1

The fourth column represents the unrestricted maximum likelihood estimate of the cumulative distribution function. As can be seen, the responses to the price of \$10 violate the monotonicity assumption for a standard distribution function: $F_{\$10} < F_{\$5}$. Pooling the \$10 and \$5 responses results in the Turnbull distribution and probability mass point estimates reported in the last two columns. Other than the \$10 responses, all other sub-samples satisfy the monotonicity assumption. Using the definition in equation (3.29), we calculate $E_{LB}(WTP)$

$$\begin{aligned} \sum_{j=0}^{M^*} t_j f_{j+1}^* &= 0 \cdot 0.343 + 5 \cdot 0.228 + 25 \cdot 0.011 + 50 \cdot 0.218 \\ &= +100 \cdot 0.075 + 200 \cdot 0.017 + 300 \cdot 0.111 \\ &= \$56.50. \end{aligned}$$

The variance is given by $V(E_{LB}(WTP))$

$$\begin{aligned} &= \sum_{j=1}^{M'} \frac{F_j^*(1 - F_j^*)}{T_j^2} (t_j - t_{j-1})^2 \\ &= \frac{0.343 \cdot 0.657}{102} (5 - 0)^2 + \frac{0.568 \cdot 0.432}{0.579 \cdot 0.421} (25 - 5)^2 + \\ &\quad \frac{0.95}{0.872 \cdot 0.128} (50 - 25)^2 + \frac{133}{(200 - 100)^2} + \frac{0.889 \cdot 0.111}{94} (300 - 200)^2 \\ &= \$29.52. \end{aligned}$$

For this example, the mean lower bound willingness to pay is \$56.50 and with a standard error of 5.50. The 95% confidence interval for lower bound WTP is \$6.50–\$5.50 (\$45.71, \$67.28). One of the advantages of the $E_{LB}(WTP)$ is the ease of constructing a confidence interval or performing hypothesis tests, because of the asymptotic normality.

3.5 A Distribution-Free Estimate of WTP

The lower bound estimator assumes that the full mass of the distribution function falls at the lower bound of the range of prices for each mass point. For example, if the probability that willingness to pay falls between t_1 and t_2 is estimated to be 0.25, then for purposes of calculating the lower bound estimate of WTP , the full 25% of the distribution function is assumed to mass at t_1 . There are methods for interpolating between price points to describe the distribution between prices simpler of such methods is a simple linear interpolation between prices used by Kristrom (1990). Instead of assuming a mass point at the lower end of the price range, we can assume that the distribution function is piece-wise linear between price points. Assuming the survivor function is piece-wise linear between prices makes the calculation of the area under the survivor function a matter of geometry. The survivor function between any two prices t_j and t_{j+1} forms a trapezoid with area equal to

$$\begin{aligned} \int_{t_j}^{t_{j+1}} (1 - F_w(w)) dw &= (1 - F_{j+1}^*) (t_{j+1} - t_j) \\ &\quad + \frac{[F_{j+1}^* - F_j^*]}{2} (t_{j+1} - t_j) \\ &= (t_{j+1} - t_j) \left(1 - \frac{(F_j^* + F_{j+1}^*)}{2} \right). \end{aligned} \quad (3.34)$$

The right hand term in equation (3.34) shows that the piece-wise linear estimator for expected willingness to pay assumes that willingness to pay is distributed uniformly between prices with the probability of a yes equal to the mid-point of the estimated probabilities at the two prices. For example, if 25% respond 'yes' to \$5 and 20% respond 'yes' to \$10 then the probability that WTP is less than any value between \$5 and \$10 is assumed to be 22.5%. Summing over all offered prices yields the estimate of expected willingness to pay

$$\begin{aligned} E(WTP) &= \sum_{j=0}^M \int_{t_j}^{t_{j+1}} [1 - F_w(w)] dw \\ &= \sum_{j=0}^M (t_{j+1} - t_j) \left(1 - \frac{(F_j^* + F_{j+1}^*)}{2} \right). \end{aligned} \quad (3.35)$$

Two problems occur in calculating expected willingness to pay in this manner: $t_0 = 0$ and $t_{M+1} =$ upper bound on WTP are not offered prices, and as such, the survivor functions at these prices are undefined. If WTP is assumed to be non-negative then the survivor function (distribution function) goes to one (zero) at a price of zero. Unless one has some insight that WTP can be less than zero, it is sensible that the probability of willingness to pay being less than zero is zero. Also, by definition, the survivor function can be defined to go to zero at the upper bound on willingness to pay. However, unless a price is offered such that all respondents say no to the highest price, any assumed upper bound for willingness to pay will be arbitrary. Differentiating equation (3.35) with respect to t_{M+1} yields the marginal change in expected willingness to pay for a one unit increase in the upper bound:

$$\frac{\partial E(WTP)}{\partial t_{M+1}^*} = \frac{1 - F_M^*}{2}. \quad (3.36)$$

As the arbitrary upper bound is increased by one dollar, the measure of expected willingness to pay will always increase. On the other hand, the lower bound estimate is independent of the upper bound. A respondent who answers 'yes' to the highest bid is assumed to have WTP equal to the highest bid. Where the piece-wise linear estimate can provide a point estimate of expected WTP , the lower bound estimate provides a more conservative estimate independent of ad hoc assumptions about the upper tail.